COMPARITIVE STUDY ON ARITHMETIC-GEOMETRIC (AG) INDEX AND GEOMETRIC-ARITHMETIC (GA) INDEX FOR VARIOUS GRAPHS

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Abstract

Topological indices are numerical quantities of a graph that are invariant under graph isomorphism. Various topological indices are categorized based on their degree, distance, and spectrum. In this paper, the comparative study is done between arithmetic- geometric (AG) index and geometric – arithmetic (GA) index which are degree-based topological indices.

Keywords: *Graphs, Topological index, arithmetic- geometric index and geometric - arithmetic index.*

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1. INTRODUCTION

A graph G consists of a finite nonempty set $V = V(G)$ of n vertices together with a prescribed set E of m unordered pairs of distinct vertices of V. Each pair $e = \{u, v\}$ of vertices in E is an edge of G, u and v joined by e. The order of G, denoted by $|V(G)| = n$, is the number of vertices in G. The size of G, denoted by $|E(G)| = m$, is the number of edges in G. If vertex set and edge set of G are finite, then G is finite. A finite graph G having no loops or multiple edges is called a simple graph. [5]

A graph G is called a bipartite graph if the vertex set V can be partitioned into two subsets V_1 and V_2 such that every edge of G joins a vertex of V_1 with a vertex of V_2. Furthermore, if every vertex of V_1 is joined to every vertex of V_2, then G is a complete bipartite graph. The complete bipartite graph with two partite sets V_1 and V_2 of vertices such that $|V_1| = a$, and $|V_2| = b$ is denoted by $K_{a,b}$. [5]

A cycle on $n \geq 3$ vertices is a closed path and is denoted by C_n. The length of a $v_0 - v_r$ walk is the number of occurrences of edges in it. [5]

The degree of a vertex v of G, denoted by $d(v)$ or $\deg(v)$, is the number of edges incident to v. A graph G is said to be regular if every vertex in G has the same degree.

Moreprecisely, G is said

Let G_1 and G_2 be two graphs with disjoint vertex sets V_1 and V_2, and edge sets E_1 and E_2, respectively. Then

1. their union $G_1 \cup G_2$ is the graph having vertex set $V_1 \cup V_2$ and the edge set $E_1 \cup E_2$.
2. their join $G_1 + G_2$ is the graph consisting of $G_1 \cup G_2$ with all edges joining V_1 with V_2.

One of the most investigated categories of topological indices used in mathematical chemistry is degree-based topological indices, which are defined in terms of the degrees of the vertices of a graph. The topological index for a graph is defined in [4],

$$TI(G) = \sum_{pq \in G} F\left(d(p), d(q)\right)$$

The graphs considered in this paper are all finite, non-trivial, undirected and connected simple graph. As usual n and m denote the number of vertices and edges of a graph G. Any undefined term or notation in this paper can be found in [1,2, 5].

Arithmetic-geometric index of a graph G is $AG(G)$ is defined by V. S. Shegehalli defined byet.al. in [7] as,

$$AG(G) = \sum_{pq \in E(G)} \left(\frac{\left(d_p + d_q\right)}{2\sqrt{\left(d_p \cdot d_q\right)}} \right).$$

Geometric -arithmetic index of a graph G is defined by $GA(G)$ is defined by D. Vukicevic et.al. in [9] as,

$$GA(G) = \sum_{pq \in E(G)} \left(\frac{2\sqrt{\left(d_p \cdot d_q\right)}}{\left(d_p + d_q\right)} \right).$$

2. COMPARITIVE STUDY BETWEEN (AG) INDEX AND (GA) INDEX :

In this section, the comparative study is made between (AG) index and (GA) index for regular graph, cycle graph, complete graph, complete bipartite graph and join of graphs.

Theorem 2.1: For a K-regular graph, the (AG) index and (GA) index are equal to size $S(G)$ of the K-regular graph.

Proof: Let G be a K-regular graph with order n. This implies the degree of every vertex in G is K. In a K-regular graph there are $\left(\frac{nK}{2}\right)$ edges. Therefore, (AG) index is obtained as,

$$AG(G) = \sum_{pq\in E(G)} \left(\frac{(d_p + d_q)}{2\sqrt{(d_p \cdot d_q)}} \right)$$

$$= \sum_{pq\in E(G)} \left(\frac{(K+K)}{2\sqrt{(K\cdot K)}} \right)$$

$$= \sum_{pq\in E(G)} \left(\frac{(2K)}{2\sqrt{(K^2)}} \right)$$

$$= \sum_{pq\in E(G)} \left(\frac{2K}{2K} \right) = \sum_{pq\in E(G)} (1)$$

$$AG(G) = \frac{nK}{2}$$

And (GA) index is obtained as,

$$GA(G) = \sum_{pq\in E(G)} \left(\frac{2\sqrt{(d_p \cdot d_q)}}{(d_p + d_q)} \right)$$

$$= \sum_{pq\in E(G)} \left(\frac{2\sqrt{(K\cdot K)}}{(K+K)} \right)$$

$$= \sum_{pq\in E(G)} \left(\frac{2\sqrt{(K^2)}}{(2K)} \right)$$

$$= \sum_{pq\in E(G)} \left(\frac{2K}{2K} \right) = \sum_{pq\in E(G)} (1)$$

$$GA(G) = \frac{nK}{2}$$

Hence, the (AG) index and (GA) index are equal to size $S(G)$ of the K-regular graph.

Example 2.1:

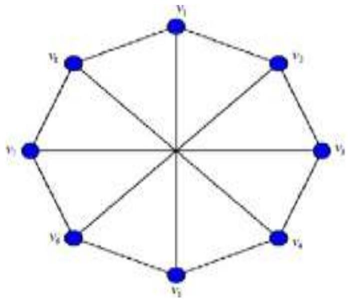

Figure 2.1: 3-Regular graph

The graph G is a 3-regular graph consisting 8 vertices. Where, $d(u_i)=3, \forall u_i \in V$ $O(G)=n=8$and $S(G)=12$. Therefore, $(AG)index=(GA)index=S(G)=12$.

Theorem 2.2: For a cycle of n-vertices, the (AG) index and (GA) index are coinciding.

Proof: Let G be a cycle of order n. This implies the degree of every vertex in G is 2. It is a 2 regular graph. Therefore, (AG) index is,

$$AG(G)=\sum_{pq\in E(G)}\left(\frac{(d_p+d_q)}{2\sqrt{(d_p\cdot d_q)}}\right)$$

$$=\sum_{pq\in E(G)}\left(\frac{(2+2)}{2\sqrt{(2\cdot 2)}}\right)$$

$$=\sum_{pq\in E(G)}\left(\frac{(4)}{2\sqrt{(4)}}\right)$$

$$=\sum_{pq\in E(G)}\left(\frac{4}{4}\right)=\sum_{pq\in E(G}(1)$$

$$AG(G)=n$$

And (GA) index is,

$$GA(G)=\sum_{pq\in E(G)}\left(\frac{2\sqrt{(d_p\cdot d_q)}}{(d_p+d_q)}\right)$$

$$=\sum_{pq\in E(G)}\left(\frac{2\sqrt{(2\cdot 2)}}{(2+2)}\right)$$

$$= \sum_{pq\in E(G)} \left(\frac{2\sqrt{(4)}}{(4)} \right)$$

$$= \sum_{pq\in E(G)} \left(\frac{4}{4} \right) = \sum_{pq\in E(G)} (1)$$

$$GA(G) = n$$

Hence, the (AG) index and (GA) index are coinciding.

Example 2.2:

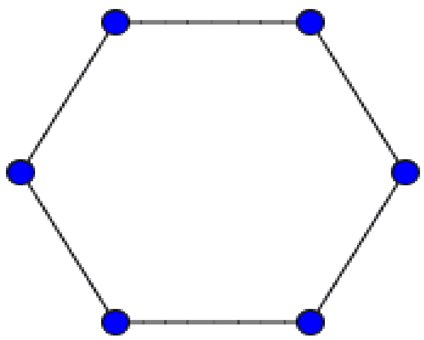

Figure 2.2: Cycle of 6 vertices

The graph G is a cycle having 6 vertices. Where, $d(u_i) = 2, \forall u_i \in V$ and $O(G) = n = 6$. Therefore, (AG) index= (GA)index $= S(G) = 6$.

Theorem 2.3: For a complete graph of n vertices, the (AG) index and (GA) index are identical.

Proof: Let G be a complete graph of order n, that is $O(G) = n$. This implies the degree of every vertex in G is $(n-1)$. In a $(n-1)$ regular graph there are, $\frac{n(n-1)}{2}$ edges.

Therefore (AG) index for complete graph is obtained as,

$$AG(G) = \sum_{pq\in E(G)} \left(\frac{(d_p + d_q)}{2\sqrt{(d_p \cdot d_q)}} \right).$$

$$AG(G) = \sum_{pq\in E(G)} \left(\frac{((n-1)+(n-1))}{2\sqrt{((n-1)^2)}} \right).$$

$$AG(G) = \sum_{pq\in E(G)} \left(\frac{(2(n-1))}{2(n-1)} \right). = \sum_{pq\in E(G)} (1).$$

$$AG(G) = \frac{n(n-1)}{2}$$

And (GA) index for complete graph is obtained as,

$$GA(G)=\sum_{pq\in E(G)}\left(\frac{2\sqrt{(d_p\cdot d_q)}}{(d_p+d_q)}\right)$$

$$GA(G)=\sum_{pq\in E(G)}\left(\frac{2\sqrt{((n-1)^2)}}{((n-1)+(n-1))}\right)$$

$$GA(G)=\sum_{pq\in E(G)}\left(\frac{2(n-1)}{2(n-1)}\right)=\sum_{pq\in E(G)}(1)$$

$$GA(G)=\frac{n(n-1)}{2}$$

Hence the (AG) index and (GA) index for complete graph are identical.

Example 2.3:

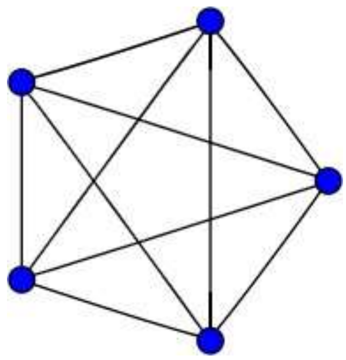

Figure 2.3: Complete graph C_5

The graph G is a complete graph of 5 vertices. Therefore, $d(u_i)=4, \forall u_i\in V$ and $O(G)=n=5$. The (AG) index and (GA) index are equal to10.

Theorem 2.4: For a complete bipartite graph $K_{m,n}$, the $(AG)index(G)=\left(\frac{\sqrt{(mn)}(m+n)}{2}\right)$ and $(GA)index(G)=\left(\frac{2(mn)^{3/2}}{(m+n)}\right)$.

Proof: Let G be a complete bipartite graph $K_{m,n}$, The graph contains two disjoint vertex sets V_m and V_n such that there exist an edge between the vertex set V_m into vertex set V_n. Therefore, the degree of every vertex in V_m and V_n is n and m respectively,

$d(v_i) = n, \forall v_i \in V_m$ and $d(v_j) = m, \forall v_j \in V_n$. There are mn edges in a complete bipartite graph $K_{m,n}$. Therefore, (AG) index

$$AG(K_{m,n}) = \sum_{pq \in E(G)} \left(\frac{(d_p + d_q)}{2\sqrt{d_p \cdot d_q}} \right)$$

$$AG(K_{m,n}) = \sum_{pq \in E(G)} \left(\frac{(m+n)}{2\sqrt{(m \cdot n)}} \right)$$

$$= \left(\frac{(m+n)}{2\sqrt{(m \cdot n)}} \right) + \left(\frac{(m+n)}{2\sqrt{(m \cdot n)}} \right) + \left(\frac{(m+n)}{2\sqrt{(m \cdot n)}} \right) + \ldots (mn) \, times$$

$$AG(K_{m,n}) = (mn) \left(\frac{(m+n)}{2\sqrt{(m \cdot n)}} \right)$$

$$AG(K_{m,n}) = \left(\frac{\sqrt{(mn)}(m+n)}{2} \right)$$

And (GA) index of a complete bipartite graph $K_{m,n}$ is ,

$$GA(K_{m,n}) = \sum_{pq \in E(G)} \left(\frac{2\sqrt{(d_p \cdot d_q)}}{(d_p + d_q)} \right)$$

$$GA(K_{m,n}) = \sum_{pq \in E(G)} \left(\frac{2\sqrt{(m \cdot n)}}{(m+n)} \right)$$

$$= \left(\frac{2\sqrt{(m \cdot n)}}{(m+n)} \right) + \left(\frac{2\sqrt{(m \cdot n)}}{(m+n)} \right) + \left(\frac{2\sqrt{(m \cdot n)}}{(m+n)} \right) + \ldots (mn) \, times$$

$$GA(K_{m,n}) = (mn) \left(\frac{2\sqrt{(m \cdot n)}}{(m+n)} \right)$$

$$GA(K_{m,n}) = \left(\frac{2(mn)^{3/2}}{(m+n)} \right)$$

Example 2.4:

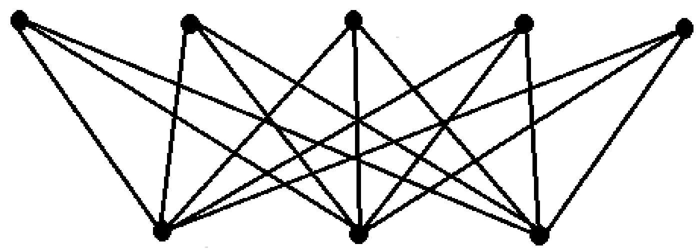

Figure 2.4: Complete bipartite graph $K_{5,3}$

The graph G is a complete bipartite graph $K_{5,3}$ having V_5 and V_3 as its vertex sets. Therefore, $d(u_i)=3, \forall u_i \in V_5$ and $d(u_j)=5, \forall u_j \in V_3$. The $(AG)index(K_{5,3}) = \left(\frac{(15)(8)}{2}\right) = 4\sqrt{15}$ and $(GA)\,\text{index}\,(K_{5,3}) = \left(\frac{(15)^{3/2}}{(4)}\right)$.

Theorem 2.5: For a join of two graphs G_1 and G_2, the (AG) index,

$$AG(G_1+G_2)=\sum_{pq\in E(G_1)}\left(\frac{(d_p+d_q+2n)}{2\sqrt{((d_p+n)\cdot(d_q+n))}}\right)+\sum_{pq\in E(G_2)}\left(\frac{(d_p+d_q+2m)}{2\sqrt{((d_p+m)\cdot(d_q+m))}}\right)$$

and (GA) index

$$GA(G_1+G_2)=\sum_{pq\in E(G_1)}\left(\frac{2\sqrt{((d_p+n)\cdot(d_q+n))}}{(d_p+d_q+2n)}\right)+\sum_{pq\in E(G_2)}\left(\frac{2\sqrt{((d_p+m)\cdot(d_q+m))}}{(d_p+d_q+2m)}\right)$$

Proof: Let a join of two graphs G_1 and G_2 be of order m and n respectively. By the definition of join of two graphs G_1 and G_2 there is an edge between every vertices in G_1 and G_2. This implies the degree of vertices in (G_1+G_2) are $(d(v_i)+n), \forall v_i \in V_1 \; and \; (d(v_j)+m), \forall v_j \in V_2$.

The, (AG) index for join of two graphs G_1 and G_2 is,

$$AG(G_1+G_2)=\sum_{pq\in E(G)}\left(\frac{(d_p+d_q)}{2\sqrt{(d_p\cdot d_q)}}\right)$$

$$=\sum_{pq\in E(G_1)}\left(\frac{(d_p+n+d_q+n)}{2\sqrt{((d_p+n)\cdot(d_q+n))}}\right)\sum_{pq\in E(G_2)}\left(\frac{(d_p+m+d_q+m)}{2\sqrt{((d_p+m)\cdot(d_q+m))}}\right)$$

$$AG(G_1+G_2)=\sum_{pq\in E(G_1)}\left(\frac{(d_p+d_q+2n)}{2\sqrt{((d_p+n)\cdot(d_q+n))}}\right)+\sum_{pq\in E(G^2)}\left(\frac{(d_p+d_q+2m)}{2\sqrt{((d_p+m)\cdot(d_q+m))}}\right)$$

And (GA) index for join of two graphs G_1 and G_2 is,

$$GA(G_1+G_2)=\sum_{pq\in E(G)}\left(\frac{2\sqrt{d_p\cdot d_q}}{\left(d_p+d_q\right)}\right)$$

$$=\sum_{pq\in E(G_1)}\left(\frac{2\sqrt{\left(\left(d_p+n\right)\cdot\left(d_q+n\right)\right)}}{\left(d_p+n+d_q+n\right)}\right)+\sum_{pq\in E(G_2)}\left(\frac{2\sqrt{\left(\left(d_p+m\right)\cdot\left(d_q+m\right)\right)}}{\left(d_p+m+d_q+m\right)}\right)$$

$$GA(G_1+G_2)=\sum_{pq\in E(G_1)}\left(\frac{2\sqrt{\left(\left(d_p+n\right)\cdot\left(d_q+n\right)\right)}}{\left(d_p+d_q+2n\right)}\right)+\sum_{pq\in E(G_2)}\left(\frac{2\sqrt{\left(\left(d_p+m\right)\cdot\left(d_q+m\right)\right)}}{\left(d_p+d_q+2m\right)}\right).$$

3. CONCLUSION

In this paper, the comparative study is done between (AG) index and (GA) index for some standard graphs such as, regular graph, cycle graph, complete graph, complete bipartitegraph and join of graphs.

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